

Differential Evolution for Optimal Power Economic/Emission Dispatch Problem with or without Considering Transmission Losses

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Abstract—This paper presents a differential evolution (DE) for optimal power economic/emission dispatch (PEED) problem. DE can efficiently search and actively explore solutions. The multiplier updating (MU) is introduced to avoid deforming the augmented Lagrange function and resulting in difficulty to solution searching. To handle the constrained optimal problem (COP), the ϵ -constraint technique is employed. The proposed approach integrates the ϵ -constraint technique, the DE and the MU. The proposed method (DE-MU) has the merit of automatically adjusting the randomly given penalty to a proper value and requiring only a small-size population. Two numerical results indicate that the proposed approach is superior to previous methods in solution quality for optimal PEED problems.

Keywords—Differential evolution, economic/emission dispatch, multiplier updating.

I. INTRODUCTION

Economic dispatch provides an avenue to power generators to provide electricity at a minimum cost. Initially, cost was the main variable considered in economic dispatch problem [1]. With the advent of environmental regulations, power generating unit emissions were introduced and used as part of the cost function for economic dispatch. Power economic/emission dispatch became then a constrained optimal problem (COP) to minimize the cost of generation, while satisfying the equality and inequality constraints of the power system and keeping pollution within limits [2-4].

Many research efforts were made for the COP [5-13]. Niknam et al. [5] proposed an innovative tribe-modified differential evolution (Tribe-MDE) for the COP. Rao and Vaisakh [6] provided a multi objective optimization approach based on adaptive clonal selection algorithm (ACSA) to solve the complex COP of thermal generators in power system. Zhang et al. [7] presented a multi-objective optimization algorithm, called the bare-bones multi-objective particle swarm optimization (BB-MOPSO) for solving the COP. Niknam and Mojarrad [8] developed a modified adaptive Θ -particle swarm optimization (MA Θ -PSO) algorithm to investigate the COP. Gong et al. [9]

described a hybrid multi-objective optimization algorithm based on PSO and DE (MO-DE/PSO) for solving the COP. Agrawal et al. [10] used a fuzzy clustering-based particle swarm (FCPSO) method to solve the COP. A strength pareto evolutionary algorithm (SPEA) based approach was employed to handle system constraints of the COP [11]. A modified harmony search algorithm (MHSA) [12] and an artificial bee colony algorithm with dynamic population-size (ABCDP) [13] were used for power optimal economic/emission problems. Storn and Price [14] developed the DE which immediately gained popularity as a robust evolutionary algorithm. DE has been widely applied to the optimization problems [5, 15-20]. Throughout the years, DE has been used extensively for optimization problems, many results of which are the best compared to other standard methodologies.

II. SYSTEM FORMULATION

In the COP formulation, these are economy and environmental impacts.

A. Economy objective F_1

The economy objective F_1 of generator power output P_i is represented as [11];

$$F_1 = \sum_{i=1}^{N_g} a_i P_i^2 + b_i P_i + c_i \quad \$/\text{h} \quad (1)$$

Were F_1 is the total cost of generation, P_i is the generation of the i th generator, a_i , b_i and c_i are coefficients of the cost curve of the i th generator, and N_g is the total number of the generators.

B. Environmental objective F_2

The emission of sulfur dioxide, nitrogen oxides, carbon monoxide gases etc., which cause atmospheric hazards, can be mathematically modeled as [11];

$$F_2 = 10^{-2}(\alpha_i + \beta_i P_i + \gamma_i P_i^2) + \xi_i e^{(\zeta_i P_i)} \quad (2)$$

Were α , β , γ , ξ , and ζ are coefficients of generator emission characteristics.

C. System constraints

To ensure a real power balance, an equality constraint is imposed:

$$\sum_{i=1}^{N_g} P_i - P_D - P_{loss} \quad (3)$$

Where P_D is the total demand, and P_{loss} is the real power loss in the transmission lines. The inequality constraint imposed on generator output is

$$P_{i\ min} \leq P_i \leq P_{i\ max} \quad (4)$$

Where $P_{i\ min}$ and $P_{i\ max}$ are the minimum and maximum limits on the loadings of the i th generator. Aggregating equations (1) to (4), the multi-objective optimization problem is formulated as;

$$\begin{aligned} & \underset{P_i}{\text{minimize}} \quad [F_1(P_i), F_2(P_i)] \\ & \text{subject to} \quad \sum_{i=1}^{N_g} P_i - P_{loss} = P_D \\ & \quad P_{i,\min} \leq P_i \leq P_{i,\max}; \quad i = 1, 2, \dots, N_g \end{aligned} \quad (5)$$

Where $F_1(P_i)$, $F_2(P_i)$ are the objective functions to be minimized over the set of admissible decision vector P_i .

III. THE PROPOSED ALGORITHM

This section describes the proposed approach for the COP. The ε -constraint technique is first provided, a brief overview of the DE is secondly discussed, then the MU is presented, and the solution procedure of the proposed approach is stated last.

A. The ε -Constraint Technique

The ε -constraint method is used to generate pareto-optimal solutions to the multi-objective problem. To proceed, one of the objective functions constitutes the primary objective function and all other objectives act as constraints. To be more specific, this procedure is implemented by replacing one objective in the problem as defined by (5) with one constraint. Reformulate the problem as follows:

$$\begin{aligned} & \min F_j(P_i), \quad j = 1 \text{ or } 2, \quad i = 1, 2, \dots, N_g \\ & \text{subject to } F_k(P_i) \leq \varepsilon_k; \quad k = 1 \text{ or } 2, \quad \text{and } k \neq j \\ & \quad \sum_{i=1}^{N_g} P_i - P_{loss} = P_D \\ & \quad P_{i,\min} \leq P_i \leq P_{i,\max}; \quad i = 1, 2, \dots, N_g \end{aligned} \quad (6)$$

B. The DE

The DE algorithm is one of the population-based optimization algorithms. The steps for implementing DE are as follows [14]:

Step 1: Initial population: A population of N_p initial solutions randomly distributed in the n_c dimensional search space of the optimization problem, are initiated. The DE uses N_p vectors of variables x in the optimization problem, namely, $x^G = \{x_i^G, i = 1, \dots, N_p\}$, as a population in generation G . For convenience, the decision vector, x_i is represented as $(x_{i1} \dots x_{ji} \dots x_{n_c i})$. Here, the decision variable, x_{ji} is directly coded as a

real value within its bounds of (x_j^{\min}, x_j^{\max}) . Each individual is generated as follows:

$$\begin{aligned} x_{ji}^G|_{G=0} &= x_j^{\min} + \text{rand}(0,1) * (x_j^{\max} - x_j^{\min}) \quad (7) \\ & j = 1, 2, \dots, n_c; \quad i = 1, 2, \dots, N_p \end{aligned}$$

()Where $\text{rand}(0, 1)$ is a random number between 0 and 1.

Step 2: Mutation operator: In mutation step, for each individual x_i (target vector) of the new population, three different individuals x_{r1} , x_{r2} , and x_{r3} ($r1 \neq r2 \neq r3 \neq i$) are pseudo-randomly extracted from the population to generate a new vector as:

$$Z_{ji} = x_{jr1} + F * (x_{jr2} - x_{jr3}), \quad j=1,2,\dots,n_c \quad (8)$$

Where $F \in [0,2]$ is a uniformly distributed random number which controls the length of the population exploration vector $(x_{r2} - x_{r3})$.

Step 3: Crossover operator: After mutation step, the crossover operator, according to the following equation, is applied on the mutation vector Z_{ji} and the vector x_{ji} to generate the trial vector U_{ji} for increasing population diversity of the mutation vector.

$$U_{ji} = \begin{cases} z_{ji}, & \text{if } \text{rand}(0,1) \leq CR \\ x_{ji}, & \text{otherwise} \end{cases} \quad (9)$$

$$j = 1, 2, \dots, n_c, \quad i = 1, 2, \dots, N_p$$

Where $CR \in [0,1]$ is known as the crossover rate which is a constant.

Step 4: Selection & evaluation operator: The selection & evaluation process is repeated for each pair of target/trial vectors using the evaluation function $F(U_{ji})$ to compare with the evaluation function value $F(x_{ji})$, and the better one will be selected to be a member of the DE population generation for the next iteration (x_{ji}^{G+1}).

C. The Multiplier Updating (MU)

Considering the nonlinear problem with general constraints as follows:

$$\begin{aligned} & \min f(x) \\ & \text{subject to } h_k(x) = 0, \quad k = 1, \dots, m_e \\ & \quad g_k(x) \leq 0, \quad k = 1, \dots, m_i \end{aligned} \quad (10)$$

Where x represents a n_c -dimensional variable, and the $h_k(x)$ and $g_k(x)$ stand for equality and inequality constraints, respectively. The augmented Lagrange function (ALF) [21] is combined with the Lagrange function and penalty terms, yielding,

$$\begin{aligned} L_a(x, \nu, \nu) &= f(x) + \sum_{k=1}^{m_e} \alpha_k \{ [h_k(x) + \nu_k]^2 - \nu_k^2 \} \\ & \quad + \sum_{k=1}^{m_i} \beta_k \{ \langle g_k(x) + \nu_k \rangle_+^2 - \nu_k^2 \} \end{aligned} \quad (11)$$

Where α_k and β_k are the positive penalty parameters, and the corresponding Lagrange multipliers $v=(v_1, \dots, v_{m_e})$ and $\nu=(\nu_1, \dots, \nu_{m_i}) \geq 0$ are associated with equality and inequality constraints, respectively. The contour of the ALF does not change shape between generations while constraints are linear. Therefore, the contour of the ALF is simply shifted or biased in relation to the original objective function, $f(x)$. Consequently, small penalty parameters can be used in the MU. However, the shape of contour of L_a is changed by penalty parameters while the constraints are nonlinear, demonstrating that large penalty parameters still create computational difficulties. Adaptive penalty parameters of the MU are employed to alleviate the above difficulties. More details of the MU have shown in [22, 23].

searches the optimal solution in the economic dispatch process and the MU effectively tackles system constraints.

IV. SYSTEM SIMULATIONS

In this section, the proposed DE-MU is applied to the standard IEEE 30-bus 6-generator test system for solving the COP by case1 without considering the transmission loss and case2 with considering the transmission loss, respectively. The detailed data of this test system are given in [11]. The proposed approach solves COP considering system constraints of power balance (3) and capacity limits (4). The MU algorithm was used in DE to handle the equality and inequality constraints. The computation was implemented on a personal computer (Intel(R) Core(TM) i9 CPU @ 3.4 GHz with 16G Ram) in FORTRAN-90 language. Setting factors used in this test are follows; the population size N_p is set as 5. The iteration numbers of outer loop and inner loop are set to (outer, inner) as (10, 3000). The implementation of the proposed algorithm for this test can be described as follows:

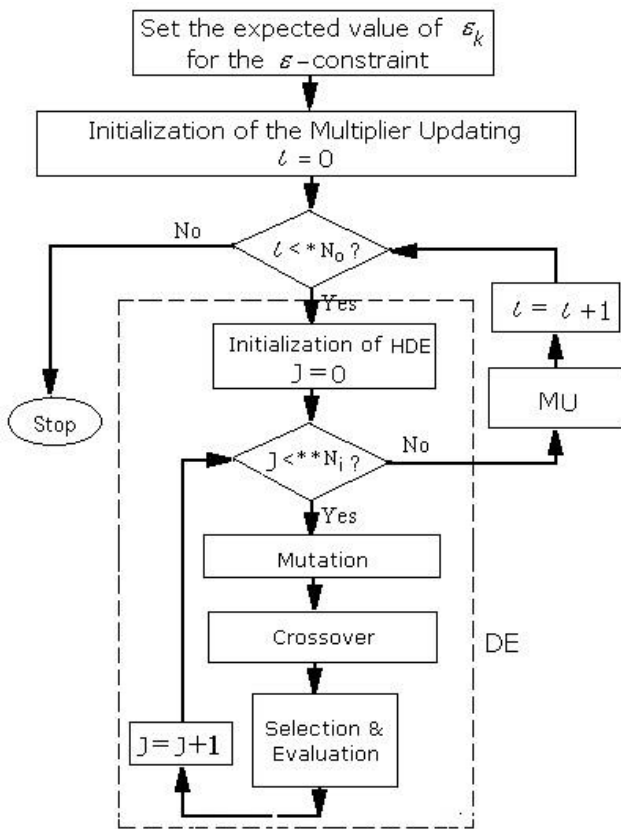


Figure 1. The flow chart of the proposed DE-MU

- *N_o : maximum number of iterations of outer loop
- **N_i : maximum number of iterations of inner loop

D. The Proposed DE-MU

Figure 1 displays the flow chart of the proposed algorithm, which has two iterative loops. The ALF is used to obtain a minimum value in the inner loop with the given penalty parameters and multipliers, which are then updated in the outer loop toward producing an upper limit of L_a . When both inner and outer iterations become sufficiently large, the ALF converges to a saddle-point of the dual problem [21]. Advantages of the proposed DE-MU are that the DE efficiently

$$L_a(P_i, v, \nu) = f_1(P_i) + \alpha_1 \{ [h_1(P_i) + \nu_1]^2 - \nu_1^2 \} + \beta_1 \{ \langle g_1(P_i) + \nu_1 \rangle_+^2 - \nu_1^2 \} \quad (12)$$

$$h_1 : P_D - \sum_{i=1}^{N_g} P_i - P_{loss} = 0 \quad (13)$$

$$g_1 : F_2(P_i) - \varepsilon_2 \leq 0 \quad (14)$$

Where h_1 stands the violation of power balance constraint (3), and g_1 stands the violation of emission objective for expected ε_2 , ($\varepsilon_2 \in [F_2^{\min}, F_2^{\max}] = [0.1942, 0.2215]$) [11]. The augmented Lagrange function (10) is solved by the proposed approach. Since cost and emission are of conflicting nature, the value of objective F_2 will be the maximum when the value of F_1 objective is the minimum and vice versa. So, the values of the best cost with F_2^{\max} and the minimum emission with F_2^{\min} are obtained by performing the ALF (12) separately. The best compromise indicates the minimum cost (the optimal power economic dispatch) within expected ε_2 . The expected ε_2 is set as F_2^{\min} for both cases1 and 2.

A. Case 1: without considering the transmission losses

The purpose of case 1 is to demonstrate that the proposed DE-MU for the optimal power economic dispatch problem without considering the transmission loss. For comparison with previous reports, Table 1 compares eight computational results obtained from the proposed DE-MU, Tribe-MDE [5], ACSA [6], BB-MOPSO [7], MAO-PSO [8], MO-DE/PSO [9], FCPSO [10], and SPEA [11]. As seen from the best solution of DE-MU listed in Table 1, the emission output is 0.1942

ton/h. It is observed that the best total cost (TC) utilizing DE-MU is 637.945142 \$/h, which is much less than the best results previously reported in FCPSO [10] and SPEA [11]. The equality constraint (13) of power balance and the expected emission limit (14) are fully satisfied. Therefore, the result obtained from

the proposed DE-MU is an optimal and feasible solution, and Table 1 demonstrates that the proposed approach is superior to previous methods in solution quality.

TABLE 1. CASE1-COMPUTATIONAL RESULTS OBTAINED FROM THE PROPOSED DE-MU AND PREVIOUS METHODS WITHOUT CONSIDERING TRANSMISSION LOSSES

Items Methods	The proposed method	ACSA [6]	BB-MOPSO [7]	MO-DE/PSO [9]	Tribe-MDE [5]	MA Θ -PSO [8]	FCPSO [10]	SPEA [11]
$P(G_1)$	0.404501	0.405160	0.4071	0.4061	0.406074	0.406074	0.4097	0.4116
$P(G_2)$	0.458192	0.458324	0.4591	0.4581	0.459069	0.459069	0.4550	0.4532
$P(G_3)$	0.538343	0.538468	0.5374	0.5408	0.537939	0.537939	0.5363	0.5329
$P(G_4)$	0.385334	0.382954	0.3838	0.3822	0.382953	0.382953	0.3842	0.3832
$P(G_5)$	0.538343	0.538726	0.5369	0.5376	0.537939	0.537939	0.5348	0.5383
$P(G_6)$	0.509287	0.510369	0.5098	0.5091	0.510027	0.510027	0.5140	0.5148
$\Sigma P(G)$	2.834000	2.834001	2.8341	2.8339	2.834001	2.834001	2.8340	2.8340
Emission (ton/h)	0.1942	0.1942	0.194203	0.194203	0.19420294	0.194202938	0.1942	0.1942
TC (\$/h)	637.945142	638.2026	638.262	638.270	638.273438	638.2734405	638.3577	638.5100

TABLE 2. CASE2-COMPARES RESULTS OBTAINED FROM THE PROPOSED DE-MU AND PREVIOUS METHODS WITH CONSIDERING TRANSMISSION LOSSES

Items Methods	The proposed method	ACSA [6]	MHSA [12]	MO-DE/PSO [9]	ABCDP [13]	Tribe-MDE [5]	BB-MOPSO [7]	MA Θ -PSO [8]
$P(G_1)$	0.401494	0.409849	0.410864	0.4118	0.410177	0.410925	0.4103	0.410925
$P(G_2)$	0.458598	0.463518	0.46203	0.4616	0.463689	0.463668	0.4661	0.463668
$P(G_3)$	0.545402	0.544375	0.547546	0.5435	0.544481	0.544419	0.5432	0.544419
$P(G_4)$	0.407396	0.388961	0.395385	0.3922	0.390432	0.390374	0.3883	0.390374
$P(G_5)$	0.544869	0.543267	0.542193	0.5454	0.544513	0.544459	0.5447	0.544459
$P(G_6)$	0.510783	0.51516	0.511229	0.5148	0.51552	0.515485	0.5168	0.515485
$\Sigma P(G)$	2.868542	2.86513	2.869247	2.869300	2.868812	2.869330	2.8694	2.869330
Loss	0.034542	0.03113	0.03519	0.03535	0.034815	0.03533	0.03537	0.03533
Emission (ton/h)	0.1942	0.19418	0.1941	0.194179	0.1942	0.194179	0.194179	0.194179
TC (\$/h)	643.8736	645.2983	645.617	646.0243	646.045	646.207003	646.4847	649.207004

B. Case 2: with considering the transmission loss

The same IEEE 30-bus 6-generator test system for solving the COP as case1 was used in case 2, except that case 2 considers the transmission loss. More comparative results are also listed in Table 2.

Table 2 compares many computational results obtained from the proposed DE-MU, Tribe-MDE [5], ACSA [6], BB-MOPSO [7], MA Θ -PSO [8], MO-DE/PSO [9], MHSA [12], and ABCDP [13]. As seen from the best

solution of the proposed DE-MU listed in Table 2, the emission output is also 0.1942 ton/h. It is observed that the best total cost (TC) utilizing by the proposed DE-MU is 643.8736 \$/h, which is much less than the best results previously reported in BB-MOPSO [7] and MA Θ -PSO [8]. The equality constraint (13) of power balance and the expected emission limit (14) are fully satisfied. Therefore, the result obtained from the proposed DE-MU is an optimal and feasible solution, and Table 2 has shown that the proposed method is

superior to previous approaches in solution quality with considering the transmission loss.

V. CONCLUSION

The proposed DE-MU for solving the COP has been presented herein. The DE helps the proposed method efficiently search and refined exploit. The MU helps the proposed method avoid deforming the ALF and resulting in difficulty of solution searching. The proposed algorithm integrates the ϵ -constraint technique, the DE and the MU that has the merit of taking a wide range of penalty parameters and a small population. The IEEE 30-bus 6-generator system is used to compare the proposed DE-MU with previous methods. Simulation results show that the proposed algorithm is superior to previous approaches in solution quality for solving the COP. The contributions of this study are the MU effectively handles system constraints of COP in emission management, the DE efficiently searches the optimal solutions for COP in the economic /emission dispatch process of power systems.

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